Perturbative Renormalizability of Chiral Two Pion Exchange and Power Counting in Nucleon-Nucleon Scattering

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Abstract. We show how to renormalize chiral two pion exchange perturbatively if one pion exchange has already been fully iterated at leading order. This particular choice corresponds to the implementation of the counting proposal of Nogga, Timmermans and van Kolck at subleading orders. We illustrate why the perturbative treatment of the two pion exchange contributions is mandatory in order to avoid certain inconsistencies in Weinberg's counting. In addition, renormalizability implies modifications of the power counting which we explore for the particular case of the singlet channel.

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INTRODUCTION

The effective field theory (EFT) formulation of nuclear forces [1, 2, 3, 4] tries to provide a consistent understanding of nuclear physics in terms of chiral symmetry, the main low energy manifestation of quantum chromodynamics. The applicability of EFT techniques relies on the existence of a separation of scales in the nuclear force: the long distance physics is known to be dominated by pion exchanges (see for example [5] for a demonstration), which in turn are constrained by the requirements of chiral symmetry. On the contrary, the nature of the interaction at short distances is poorly understood, and has been traditionally treated in a purely phenomenological manner. However, the specific parametrization used for the short range physics is inessential for the description of low energy phenomena.

In Weinberg's original formulation [6, 7], which represents the first proposal for constructing an EFT of nuclear forces, the nuclear potential is expanded as a power series (or power counting) in terms of the low energy scales of the system, such as the pion mass or the nucleon momentum. The resulting chiral potential is inserted into the Schrödinger or Lippmann-Schwinger equation, from which wave functions and observables can be computed. This prescription takes into account the non-perturbative nature of nuclear forces and fits naturally into the traditional paradigm of nuclear physics. The Weinberg approach is phenomenologically very successful, as exemplified by the N^{3}LO calculations of Refs. [8, 9]. From a theoretical point of view, however, the previous calculations are rather unsatisfactory in the sense that the cut-off is fined tuned inside a narrow window. We expect any EFT calculation to be fairly cut-off independent, a prospect which has encouraged the search for alternatives to Weinberg,

like the KSW counting [10, 11, 12], or, more recently, non-perturbative renormalization [13, 14, 15, 16, 17, 18]. Although the requirement of renormalizability has been put in question in [19, 20] (see also Ref. [21] for a balanced discussion on the merits and disadvantages of the different approaches), we will show here that cut-off dependence is not the only problem that affects the Weinberg scheme.

In this contribution we will implement the power counting proposal of Nogga, Timmermans and van Kolck (NTvK) [13] at next-to-leading and next-to-next-to-leading order [22]. In this approach, the leading order piece of the chiral nucleon-nucleon (NN) interaction, one pion exchange, is iterated to all orders and renormalized, a requisite which implies the modification of the power counting for certain short range operators. The subleading pieces of the interaction are treated in perturbation theory, and as we will show for the particular case of the singlet channel, cut-off independence induces further modifications to the power counting [22] which were not present in the original Weinberg formulation. These modifications are compatible with the renormalization group analysis of the NTvK counting made by Birse [23]. We also discuss the role of the cut-off and the possible interpretation of regularization and renormalization in EFT.

POWER COUNTING

In Weinberg's power counting [6, 7], the NN potential is described as a low energy expansion in terms of a ratio of scales, Q/Λ_0

$$V_{\rm NN}(\vec{q}) = V_{\chi}^{(0)}(\vec{q}) + V_{\chi}^{(2)}(\vec{q}) + V_{\chi}^{(3)}(\vec{q}) + \mathcal{O}(\frac{Q^4}{\Lambda_0^4})\,, \tag{1}$$

where Q represents the low energy scales of the system, such as the pion mass m_π or the momentum of the nucleons p, and Λ_0 represents the high energy scales of the system, like the nucleon mass M_N or the rho mass m_ρ. The rules by which a certain operator or diagram is assigned a given power of Q/Λ_0 are called a power counting: in Weinberg's proposal it is implicitly assumed that naive dimensional analysis provides a good power counting for the NN potential. The potential is expected to be iterated at all orders in the Schrödinger or Lippmann-Schwinger equation.

The organization of the chiral potential is as follows: at order Q^0 or leading order (LO) the potential consists on one pion exchange (OPE) and two contact interactions. At order Q^2 or next-to-leading order (NLO), leading chiral two pion exchange (TPE) and seven derivative contact terms are added. At order Q^3 or next-to-next-to-leading order (N^{2}LO) subleading two pion exchange enters the potential. The pion piece of the potential is constrained by chiral symmetry, while the contact piece is generally used to fit parameters.

Power counting determines the convergence of the chiral expansion of the NN potential: we expect the chiral potential to converge at large distances / small momenta, that is, soft scales, and to diverge at hard scales. The previous is displayed in Fig. 1, in which we can see how at distances above $2-3\,$fm the leading order of the chiral expansion of the potential dominates, while the subleading contributions only represent a small correction to the leading order piece. In this regard, the chiral NN potentials realize the well

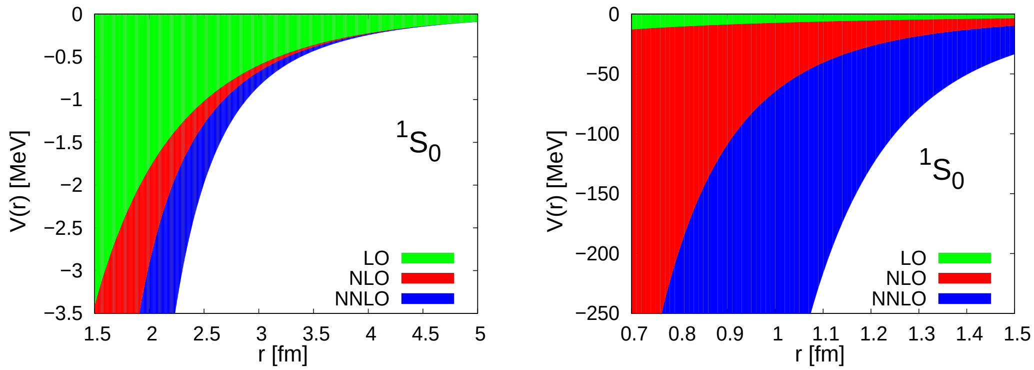

FIGURE 1. The pion (finite range) component of the chiral nucleon-nucleon potential in the $^1S_0$ channel. In the left panel (a) we show the chiral potential in the $r = 1.5 - 5.0\,\mathrm{fm}$ range: at these distances, each additional contribution to the chiral potential is suppressed due to power counting. This results in a convergent pattern for the chiral potentials at long distances. In the right panel (b), the different contributions to the chiral potential in the $r = 0.7 - 1.5\,\mathrm{fm}$ range are plotted. At short distances we see the opposite situation: higher order contributions are increasingly singular, and the chiral expansion of the nucleon-nucleon potential does not converge.

known long distance dominance of one pion exchange in traditional nuclear physics. However, as can also be seen in Fig. 1, at distances below $1-2\,\mathrm{fm}$ each additional contribution to the chiral potential is bigger than the previous one, meaning that the chiral expansion will not converge. In fact, on dimensional grounds we expect the pion piece of the order Q^ν contribution to the chiral potential to behave as

$$V^{(\nu)}_{\chi,pions}(\vec{q}) \sim \frac{|\vec{q}|^\nu}{\Lambda_0^\nu}\, f(\frac{|\vec{q}|}{m_\pi})\,, \tag{2}$$

where $\vec{q}$ is the momentum exchanged between the nucleons and $f(x)$ is some non-polynomial function, like a logarithm. Fourier-transforming the previous expression to coordinate space, we find

$$V^{(\nu)}_{\chi,pions}(\vec{r}) \sim \frac{1}{\Lambda_0^\nu\, r^{\nu+3}}\, g(m_\pi r)\,, \tag{3}$$

where $g(x)$ contains an exponential factor e^{-nx}, with n the number of pions which are exchanged between the nucleons.

The behaviour of the order Q^ν pieces of the chiral potential poses the problem of how to treat the related short range singularities. The usual way to deal with this is to apply a renormalization procedure. The recipe is (i) to regularize the short range pieces of the the potential, usually by including a cut-off in the calculations, for example $V(r) \to V(r)\,\theta(r-r_c)$ if we are working in coordinate space or $\langle p'|V|p\rangle \to \langle p'|V|p\rangle\,\theta(\Lambda - p')\,\theta(\Lambda - p)$ if we are in momentum space, and (ii) use the counterterms to absorb the undesired cut-off dependence induced by the previous regularization procedure. Then we iterate the potential in the Schrödinger or Lippmann-Schwinger equation and fix the finite part of the counterterms to fit observables. As commented in the introduction, this scheme has led to an impressive phenomenological description of NN scattering [8, 9].

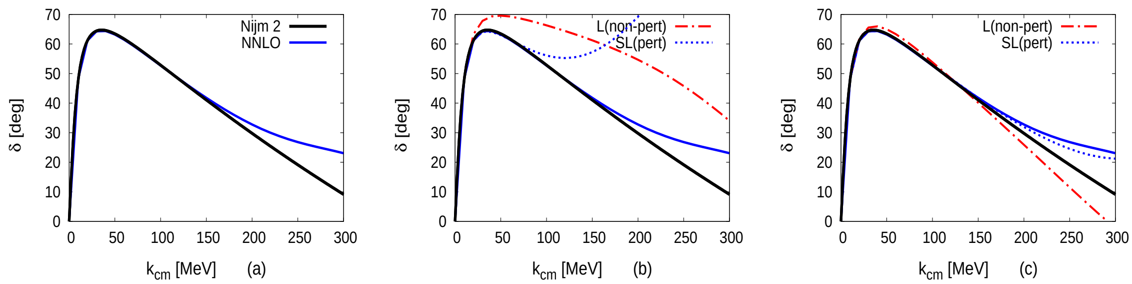

FIGURE 2. In panel (a), the phase shifts for the $^1S_0$ singlet channel computed in the (non-perturbative) Weinberg counting at N^{2}LO by fitting the counterterms to the phase shifts for $k \leq 200\,\mathrm{MeV}$. (b) The N^{2}LO phase shifts are approximated by an scheme in which the leading order (L) consists on OPE and the C_0 counterterm, while in the subleading order (SL) calculation TPE and the C_2 counterterm are added in perturbation theory. (c) In this case the N^{2}LO phase shifts are approximated at leading order by TPE and C_0 and at subleading order (perturbative) OPE and C_2.

In the panel (a) of Fig. 2 we show a particular realization of the Weinberg counting at N^{2}LO for the $^1S_0$ singlet channel. Following [24], we use $f_\pi = 92.4\,\mathrm{MeV}$, $m_\pi = 138.04\,\mathrm{MeV}$, $d_{18} = -0.97\,\mathrm{GeV}^2$, $c_1 = -0.81\,\mathrm{GeV}^{-1}$, $c_3 = -3.40\,\mathrm{GeV}^{-1}$ and $c_4 = 3.40\,\mathrm{GeV}^{-1}$. We do not implement however spectral regularization. The calculations are done by solving the Lippmann-Schwinger equation with a gaussian regulator of the type e^{-p^6/Λ^6} and a cut-off $\Lambda = 400\,\mathrm{MeV}$. The counterterms C_0 and C_2 have been fitted to reproduce the Nijmegen II phase shifts [25] for momenta $k \leq 200\,\mathrm{MeV}$.

Inconsistencies in the Weinberg Counting

There is an inherent problem in iterating the full potential in order to obtain observables. As the chiral potentials become increasingly singular at hard scales, it may be possible for the subleading pieces of the potential to dominate the amplitudes, even at moderate energies, if the cut-off is hard enough. The possibility of a power counting breakdown due to the interplay of iterations and large cut-offs was already discussed by Lepage [26] and more recently by Epelbaum and Gegelia [20]. The problem is whether the cut-offs usually taken in nuclear EFT ($\Lambda \sim 0.5\,\mathrm{GeV}$) are hard enough to trigger this undesirable situation.

The previous question can be answered by playing different games with the $^1S_0$ phase shifts of Fig. 2. From power counting we expect that the full T-matrix can be approximated by a T-matrix in which the leading piece of the interaction has been iterated and the subleading pieces have been included as perturbations:

$$T(k) = \underbrace{T^{(0)}(k)}_{\text{Non-perturbative}} + \underbrace{T^{(2)}(k) + T^{(3)}(k)}_{\text{Perturbative}} + \mathcal{O}(Q^4/\Lambda_0^4)\,. \tag{4}$$

This particular scheme is fulfilled in the panel (b) of Fig. 2, where we can see that power counting fails already at $k \simeq 100\,\mathrm{MeV}$. Of course, this is an unsatisfactory situation, but

it can get worse. In fact we can try the following *anti* power counting scheme,

$$T(k) = \underbrace{T^{(0)}(k)}_{\text{Perturbative}} + \underbrace{T^{(2)}(k) + T^{(3)}(k)}_{\text{Non-perturbative}} + \mathscr{O}(Q^4/\Lambda_0^4) \tag{5}$$

in which the subleading pieces of the interaction behave as leading order pieces. This power counting is realized in panel (c) of Fig. 2, and results in a very good approximation to the full phase shifts. That is, the previous proposal is the underlying power counting of the specific N^2LO calculation of Fig. 2 [1]. This power counting *extravaganza* is a nice example of the kind of unexpected behaviours discussed by Lepage [26]: an order Q^3 operator is behaving as being of order Q^{-1}.

PERTURBATIVE RENORMALIZABILITY OF CHIRAL TPE

Perturbative Weinberg Counting

A particular way to avoid these power counting inconsistencies is to treat the subleading order interactions in perturbation theory. If subleading contributions to the amplitude are expected to be small, it is natural to treat this pieces perturbatively. This choice, which we will call perturbative Weinberg, enforces power counting in observables independently of the cut-off. However, there are still problems with hard enough cut-offs, like poor convergence of the chiral expansion or divergences.

Perturbative Weinberg has been explored by Shukla et al. [27] for the particular case of the singlet $^1S_0$ channel, where they find that the short range physics is compatible with Weinberg in the range of coordinate space cut-offs $r_c = 1.4 - 1.8\,\mathrm{fm}$ (but fails for $r_c < 1.0\,\mathrm{fm}$). The recent lattice EFT calculations of Refs. [28, 29, 30, 31, 32] (see also D. Lee's contribution to these proceedings) also treat the subleading operators perturbatively. They employ a spatial (temporal) lattice spacing of $a_s = 1.97\,\mathrm{fm}$ ($a_t = 1.32\,\mathrm{fm}$) and provide an acceptable description of NN phase shifts for $k \leq 100\,\mathrm{MeV}$ [29] and of light nuclei [31, 32]. These results are encouraging for any perturbative setup; however, they are limited to rather soft cut-offs for good reasons.

Renormalizability and Modifications to the Power Counting

The problem with perturbative Weinberg is that the scattering amplitude diverges for hard cut-offs. In the particular case of the singlet $^1S_0$ channel, we find that only using the counterterms prescribed by Weinberg, that is $V_{\chi,\mathrm{contact}}^{(\nu=2,3)} = C_0^{(\nu)} + C_2^{(\nu)}(p^2 + p'^2)$, leads to the following divergences in the subleading pieces of the T-matrix:

$$T(\Lambda) = T^{(0)}(\Lambda) + \underbrace{T^{(2)}(\Lambda)}_{\sim \log\Lambda} + \underbrace{T^{(3)}(\Lambda)}_{\sim\Lambda} + \mathscr{O}(Q^4/\Lambda_0^4)\,. \tag{6}$$

[1] However, in most cases it is impossible to uncover any underlying power counting scheme.

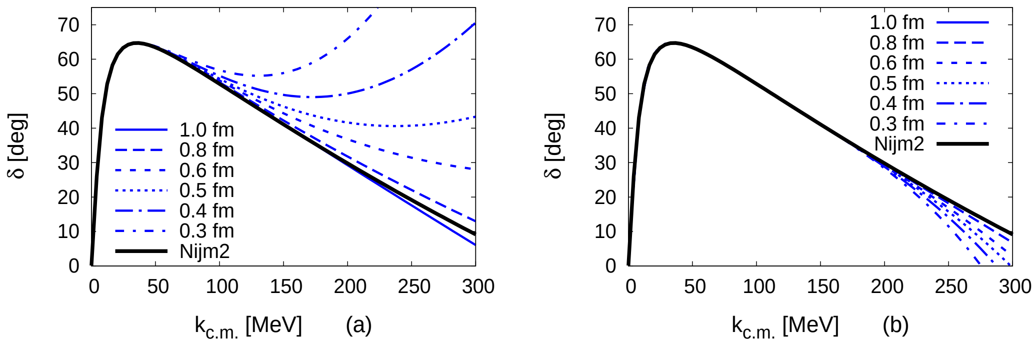

FIGURE 3. Phase shifts for the $^1S_0$ singlet channel with non-perturbative OPE and perturbative TPE at N^{2}LO. The left panel (a) shows the resulting phase shifts when only the counterterms prescribed by Weinberg counting (i.e. C_0 and C_2) are included in the calculations. In such a case the $\mathcal{O}(Q^3)$ contribution to the phase shifts diverges as k^4/r_c, being k the center of mass momentum and r_c the coordinate space cut-off. The right panel (b) shows the N^{2}LO phase shifts when the additional C_4 counterterm is included. The residual cut-off dependence of the phase shifts is now proportional to $k^6 r_c$.

In panel (a) of Fig. 3 we can see this linear divergence of the N^{2}LO (order Q^3) phase shifts for a coordinate space computation (the relation between the coordinate and momentum space cut-off is roughly $r_c \simeq \pi/2\Lambda$ [17]). The previous divergences can be cured by adding a new counterterm at NLO, that is, by taking $V_{\chi,\text{contact}}^{(\nu=2,3)} = C_0^{(\nu)} + C_2^{(\nu)}(p^2+p'^2) + C_4^{(\nu)}(p^4+p'^4)$. This is equivalent to modify the power counting rules for the C_4 operator, which is promoted from order Q^4 to order Q^2, as determined by Birse [23]. In panel (b) of Fig. 3 we can see the N^{2}LO phase shifts when the C_4 operator is included in the computations, leading to an amplitude which is free of ultraviolet divergences.

Of course this is not new: five years ago, Nogga, Timmermans and van Kolck [13] discovered by the numerical exploration of a large range of cut-off values that the renormalizability of the Lippmann-Schwinger equation for the leading order OPE potential required certain modifications to the Weinberg counting. This possibility was in fact anticipated in Ref. [14] on purely analytical grounds and can be easily understood in terms of the non-perturbative renormalization of singular interactions [15, 16, 17, 18]. In Ref. [23] Birse studied in detail the power counting resulting from iterating OPE using renormalization group analysis (RGA) techniques (see also Ref. [33] for a more informal exposition). This power counting is compatible with the short range physics extracted in Refs. [34, 35, 36] from the *deconstruction* of the phenomenological phase shifts with perturbative TPE.

Recently, in Ref. [22] the NLO and N^{2}LO phase shifts for central waves in the NTvK counting have been obtained for the first time, resulting in a good description of both the $^1S_0$ singlet and the $^3S_1-{}^3D_1$ triplet phase shifts. The power counting is determined by requiring perturbative renormalizability, as explained previously for the particular case of the singlet channel. The counting obtained in this way is basically equivalent to the

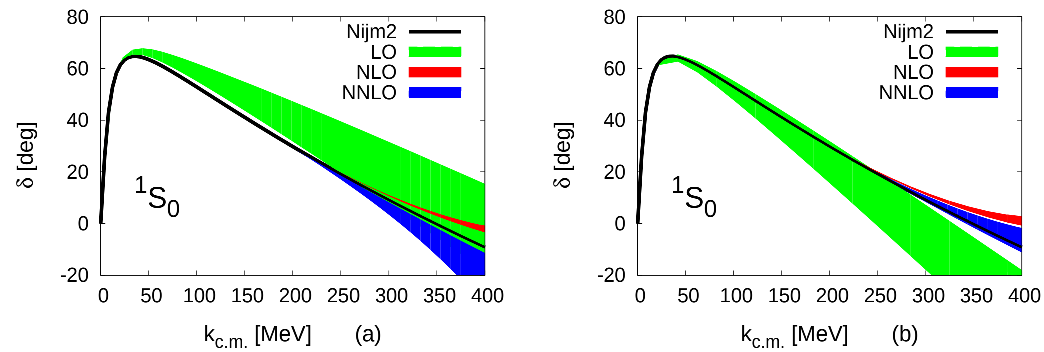

FIGURE 4. Phase shifts for the $^1S_0$ singlet channel with non-perturbative OPE and perturbative TPE at LO, NLO and N^{2}LO. The left panel (a) shows the resulting phase shifts in the Nogga, Timmermans, van Kolck counting for the cut-off range $r_c = 0.6 - 0.9\,\mathrm{fm}$. The right panel (b) shows the phase shifts for the cut-off range $r_c = 0.9 - 1.2\,\mathrm{fm}$.

one proposed by Birse [23], with some minor differences in the triplet channel [2].

In Fig. 4 we can see the N^{2}LO phase shifts in the singlet channel for the cut-off ranges $r_c = 0.6 - 0.9\,\mathrm{fm}$ and $r_c = 0.9 - 1.2\,\mathrm{fm}$. In panel (a) we follow the philosophy of Ref. [19], in which the cut-off is varied around the purported breakdown scale, probably $0.7 - 0.8\,\mathrm{fm}$, the distance at which most phenomenological potentials have their minima (signalling the point in which the short range repulsion starts to become stronger than the long range attraction). In panel (b) we follow Ref. [12], in which the cut-off is interpreted as a parameter controlling the convergence of the EFT expansion: a softer cut-off in general improves convergence. However, in both cases the phase shifts are pretty similar.

Which is the correct value for the Cut-off?

The correct value of the cut-off has become a contentious issue in nuclear EFT. Here we will first consider the problem of the cut-off from the point of view of the practical advantages that certain cut-off values entail. Then we will propose a possible interpretation.

Perturbative renormalization guarantees that the amplitudes are free of divergences when the cut-off is removed. However, there is still a serious problem with removing the cut-off: the convergence of the perturbation series is not assured for cut-off radii below $r_c \simeq 0.6 - 0.7\,\mathrm{fm}$. This is due to the appearance of the first deeply bound state, which generates a pole in the renormalization group evolution of the $C_0(r_c)$ counterterm, a feature that cannot be reproduced perturbatively. As a consequence, the related expansion for $C_0(r_c) = C_0^{(0)}(r_c) + C_0^{(2)}(r_c) + C_0^{(3)}(r_c) + \mathscr{O}(Q^4)$ must fail for $r_c < 0.6 - 0.7\,\mathrm{fm}$, spoiling the power counting.

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[2] This may be a consequence of the simplifications made in Ref. [23] for treating the coupled channels

On the other hand, if the cut-off is of the order of $1.4 - 1.8\,\mathrm{fm}$, there is no practical advantage on modifying the power counting, at least in the singlet, as the phase shifts are already well described by the perturbative Weinberg setup of Ref. [27].

In this regard, not all regularization schemes or cut-off values are adequate to realize a particular power counting proposal. Probably the best example is the KSW counting [10, 11], which can be implemented with power divergence subtraction (PDS) but not with minimal subtraction (MS). As was shown by Cohen and Hansen [37], KSW can also be realized as a cut-off theory (see also the related observations of Ref. [20]). However, independently of whether dimensional or cut-off regularization is used, there are certain limitations on the regularization scale. The analysis of the running of the C_0 counterterm of Ref. [11] requires $\mu < \Lambda_{\rm NN} \sim 300\,\mathrm{MeV}$, or equivalently $r_c > 1/\Lambda_{\rm NN}$ if the arguments are applied to a coordinate space cut-off, for the KSW counting to work.

From the previous we are tempted to interpret that the role of regularization and renormalization is to guarantee that power counting is correctly implemented. In this regard, the main inconsistency in current implementations of Weinberg is the power counting *extravaganza* phenomenon discussed earlier, which is in fact a consequence of the cut-off range employed in the calculations. This interpretation is complementary to the role suggested for the cut-off by Beane et al. [12], in which the cut-off may be chosen in order to improve the convergence of the expansion. However, as explained by Birse [35], these interpretations are unorthodox and require a firmer theoretical basis. In particular, for perturbative chiral TPE the optimal cut-off range lies probably in the vicinity of $\sim 1\,\mathrm{fm}$ or above.

CONCLUSIONS

The perturbative treatment of chiral two pion exchange provides the opportunity to construct scattering amplitudes with are compatible with the requirements of renormalizability and power counting within an effective field theory framework [22]. On the contrary, non-perturbative approaches are not guaranteed to fulfill these conditions; in particular, power counting may be lost if the cut-off is too hard, as exemplified with an example Weinberg calculation at N^{2}LO. The perturbative TPE calculations show the feasibility of the Nogga, Timmermans and van Kolck proposal [13]. While the role of the cut-off is still (and will be for some time) a controversial issue, we propose a sensible interpretation in the line of Ref. [12] which may be able to reconcile the difficult requirements for the EFT formulation of a problem with a poor separation of scales.

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REFERENCES

1. S. R. Beane, P. F. Bedaque, W. C. Haxton, D. R. Phillips, and M. J. Savage, *At the Frontiers of Particle Physics: Handbook of QCD, Vol 4, World Scientific Publishing Company* (2002).
2. P. F. Bedaque, and U. van Kolck, *Ann. Rev. Nucl. Part. Sci.* **52**, 339–396 (2002), `nucl-th/0203055`.
3. E. Epelbaum, *Prog. Part. Nucl. Phys.* **57**, 654–741 (2006), `nucl-th/0509032`.
4. E. Epelbaum, H.-W. Hammer, and U.-G. Meißner, *Rev. Mod. Phys.* **81**, 1773–1825 (2009), `0811.1338`.
5. M. C. M. Rentmeester, R. G. E. Timmermans, J. L. Friar, and J. J. de Swart, *Phys. Rev. Lett.* **82**, 4992–4995 (1999), `nucl-th/9901054`.
6. S. Weinberg, *Phys. Lett.* **B251**, 288–292 (1990).
7. S. Weinberg, *Nucl. Phys.* **B363**, 3–18 (1991).
8. D. R. Entem, and R. Machleidt, *Phys. Rev.* **C68**, 041001 (2003), `nucl-th/0304018`.
9. E. Epelbaum, W. Glöckle, and U.-G. Meißner, *Nucl. Phys.* **A747**, 362–424 (2005), `nucl-th/0405048`.
10. D. B. Kaplan, M. J. Savage, and M. B. Wise, *Phys. Lett.* **B424**, 390–396 (1998), `nucl-th/9801034`.
11. D. B. Kaplan, M. J. Savage, and M. B. Wise, *Nucl. Phys.* **B534**, 329–355 (1998), `nucl-th/9802075`.
12. S. R. Beane, D. B. Kaplan, and A. Vuorinen, *Phys. Rev.* **C80**, 011001 (2009), `0812.3938`.
13. A. Nogga, R. G. E. Timmermans, and U. van Kolck, *Phys. Rev.* **C72**, 054006 (2005), `nucl-th/0506005`.
14. M. Pavon Valderrama, and E. Ruiz Arriola, *Phys. Rev.* **C72**, 054002 (2005), `nucl-th/0504067`.
15. M. Pavon Valderrama, and E. R. Arriola, *Phys. Rev.* **C74**, 054001 (2006), `nucl-th/0506047`.
16. M. Pavon Valderrama, and E. R. Arriola, *Annals Phys.* **323**, 1037–1086 (2008), `0705.2952`.
17. D. R. Entem, E. Ruiz Arriola, M. Pavon Valderrama, and R. Machleidt, *Phys. Rev.* **C77**, 044006 (2008), `0709.2770`.
18. M. Pavon Valderrama, and E. Ruiz Arriola (2010), `1005.0744`.
19. E. Epelbaum, and U. G. Meißner (2006), `nucl-th/0609037`.
20. E. Epelbaum, and J. Gegelia, *Eur. Phys. J.* **A41**, 341–354 (2009), `0906.3822`.
21. R. Machleidt, and D. R. Entem, *J. Phys.* **G37**, 064041 (2010), `1001.0966`.
22. M. Pavon Valderrama (2009), `0912.0699`.
23. M. C. Birse, *Phys. Rev.* **C74**, 014003 (2006), `nucl-th/0507077`.
24. E. Epelbaum, W. Glöckle, and U.-G. Meißner, *Eur. Phys. J.* **A19**, 401–412 (2004), `nucl-th/0308010`.
25. V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, *Phys. Rev.* **C49**, 2950–2962 (1994), `nucl-th/9406039`.
26. G. P. Lepage (1997), `nucl-th/9706029`.
27. D. Shukla, D. R. Phillips, and E. Mortenson, *J. Phys.* **G35**, 115009 (2008), `0803.4190`.
28. B. Borasoy, E. Epelbaum, H. Krebs, D. Lee, and U.-G. Meißner, *Eur. Phys. J.* **A31**, 105–123 (2007), `nucl-th/0611087`.
29. B. Borasoy, E. Epelbaum, H. Krebs, D. Lee, and U.-G. Meißner, *Eur. Phys. J.* **A35**, 343–355 (2008), `0712.2990`.
30. E. Epelbaum, H. Krebs, D. Lee, and U.-G. Meißner, *Eur. Phys. J.* **A41**, 125–139 (2009), `0903.1666`.
31. E. Epelbaum, H. Krebs, D. Lee, and U.-G. Meißner, *Phys. Rev. Lett.* **104**, 142501 (2010), `0912.4195`.
32. E. Epelbaum, H. Krebs, D. Lee, and U.-G. Meißner, *Eur. Phys. J.* **A45**, 335–352 (2010), `1003.5697`.
33. M. C. Birse (2009), `0909.4641`.
34. M. C. Birse, *Phys. Rev.* **C76**, 034002 (2007), `0706.0984`.
35. M. C. Birse (2010), `1007.0540`.
36. K. L. Ipson, K. Helmke, and M. C. Birse (2010), `1009.0686`.
37. T. D. Cohen, and J. M. Hansen, *Phys. Lett.* **B440**, 233–238 (1998), `nucl-th/9808006`.